Exhibit E(1)

TABLE 2 **Measures of Innovation and Other Data on Computed Tomography Scanners**

Year	ΔW	TW	$R\&D$	Number of Firms	Number of New Brands	Number of New Adopters
1973	2.99	638	20.6[a]	3	1	16
1974	8.71	6926	22.6	8	1	74
1975	1.51	1503	59.7	12	4	216
1976	4.78	5959	96.1	13	11	317
1977	0.94	997	79.7	14	14	328
1978	0.12	79	64.3	11	6	211
1979	0.14	73	56.1	9	5	209
1980	0.07	30	46.4	8	2	177
1981	0.18	79	37.9	8	3	101
1982	0.20	87	37.9	8	8	•

Source: Trajtenberg (1989).
The figures for ΔW, TW, and $R\&D$ are in millions of constant 1982 dollars.
[a] This figure refers to total R&D expenditures from 1968 through 1973.

TW).[13] By indicators I formally mean predictors, and hence, the statistical criterion to be used is the mean-squared prediction error: $MSE = E(\Delta W - P)^2$. Assuming that the bivariate distribution $f(\Delta W, P)$ is such that the regression function of ΔW on P is linear (e.g., a bivariate normal), then the MSE of the (best) linear predictor is just $\sigma^2_{\Delta W}(1-\rho^2)$, where ρ^2 is the correlation coefficient between ΔW and P. (See Lindgren (1976).) Thus, in probing the adequacy of alternative patent indicators, I used the correlation coefficient as the sole criterion.[14] The hypotheses tested are (note that they refer to a given technological field, or product class, as it evolves over time) as follows:

Hypothesis 1. Patent counts weighted by citations (WPC) are good indicators of the value of innovations as measured by ΔW or TW, but simple counts (SPC) are not.

Hypothesis 2. Simple patent counts are good indicators of the inputs to the innovative process as measured by R&D expenditures.

In order to examine them, I considered two alternative patent counts: one including all patents in CT and another based on patents granted just to CT manufacturers. (The latter accounted for 66% of all patents and 80% of all citations.) Since ΔW and TW were computed on the basis of the CT scanners actually marketed in the U.S., we would expect those measures to be more highly correlated with the patents granted just to firms in CT.[15] I also considered various lags between patent counts and the measures ΔW and TW. At issue here is the expected time interval between the application date of a patent and the appearance in the market of the innovation disclosed in that patent.[16] Since the interest of inventors is to file for a patent as early as possible, the actual timing must be effectively constrained by the stringency of the applications requirements set by the Patent Office.

[13] Recalling that ΔW refers to the incremental gains, whereas TW stands for total gains, it is not clear *a priori* which of them is more relevant in the present context.

[14] Less formally, since the maintained hypothesis is that the measures $\{\Delta W, TW\}$ accurately capture the value of innovations, the only remaining question is whether patents (which could be at best just an indirect manifestation of the same) closely follow the path of those variables over time. More pragmatically, with the small number of observations available, it would have been very hard to estimate anything but simple correlations.

[15] That would not be so only if the appropriability of the patents issued to the other assignees had been extremely low, i.e., only if CT manufacturers benefitted from the innovations done by other inventors as much as they did from their own.

[16] For a detailed discussion of this issue, see Trajtenberg (1990).

Those requirements are significantly more severe in the U.S. than in Europe; in fact, 56% of all patents in CT were applied for in various European countries (primarily the U.K. and Germany) before being filed in the U.S., with an average lag between the two applications of thirteen months. Thus, we would expect the lag between patent counts and ΔW to be quite short if patents are dated according to the application date in the U.S. (as done here), and over a year longer than that (on average) if the application date abroad is taken into consideration.

The data used in testing the hypotheses consist of yearly observations on ΔW, TW, and patent counts, covering the period of 1973–1982. (See Tables 1 and 2.) As mentioned above, this is when the bulk of the innovative action in CT took place; in fact, the pace of advancement had already subsided in the late seventies, and the technology barely changed during the eighties. Thus, the fact that the period examined here is only a decade long does not impair the validity of the statistical analysis performed: the history of innovation in CT is short, and hence, there are few observations; this is, however, essentially the whole history.

□ **Testing the first hypothesis.** Table 3 presents the correlations between patent counts and $\{\Delta W, TW\}$, with the former variables lagged between zero and six months.[17] The first and most important finding is that WPC_t is, in effect, correlated with the value measures of innovation, whereas SPC_t clearly is not, in all the cases considered. Thus, the evidence

TABLE 3 **Correlations[a] of Simple and Weighted Patent Counts with ΔW, TW**

Lags	All Patents ΔW	All Patents TW	Patents to Firms in CT ΔW	Patents to Firms in CT TW
	(a) With Weighted Counts			
Contemporary	0.509 (0.13)	0.587 (0.07)	0.616 (0.06)	0.626 (0.05)
3 Months	0.513 (0.13)	0.635 (0.05)	**0.685** (0.03)	**0.755** (0.01)
4 Months	0.480 (0.16)	0.600 (0.07)	0.677 (0.03)	0.744 (0.01)
6 Months	0.317 (0.37)	0.466 (0.17)	0.495 (0.15)	0.605 (0.006)
	(b) With Simple Counts			
Contemporary	−0.162 (0.65)	0.032 (0.93)	−0.087 (0.81)	0.093 (0.80)
3 Months	−0.198 (0.58)	0.006 (0.99)	−0.076 (0.83)	0.131 (0.72)
6 Months	−0.283 (0.43)	−0.090 (0.81)	−0.175 (0.63)	0.027 (0.94)

[a] Pearson correlation coefficients.
Significance levels for H_0: corr = 0 are given in parentheses.

[17] Even though the figures are annual, the lag could be varied by monthly increments, since the patent data are virtually continuous over time. Note also that since the ΔW series begins in 1973, I just added the 1972 (first) patent to the patent count of 1973; that is, the ΔW figure for 1973 refers to the first CT scanner marketed, and hence, it obviously corresponds to the initial patents in the field, including the very first.

provides ample support for the first hypothesis. Second, the correlations increase substantially as I narrow the scope of the (weighted) counts to the patents granted to firms in CT. This implies, as suggested, that the appropriability of patents awarded to other assignees was not nil. Third, the correlations peak when the patent counts are lagged just one quarter, declining monotonically as the lag increases.[18] Superimposing the mean foreign-U.S. application lag of thirteen months mentioned earlier, one obtains an overall lead time of sixteen months. This is consistent with the intense technological rivalry that characterized the market for CT scanners in the seventies.

Returning to the basic finding of a high correlation between WPC_t and ΔW_t, I can now (re)interpret the distribution of citation counts across patents as an implied distribution of the value of innovations. As shown in Table 4, the observed distribution fits well the received wisdom on this matter (see, for example, Pakes and Schankerman (1984) and Pakes (1986)): it is very skewed, with almost half the patents never cited (and hence, of little *ex post* value) and a lucky few being worth a great deal.[19] Thus, contrary to the often-voiced view that patent data cannot possibly capture important innovations, the results here show that citation counts can span well the whole range of innovations.[20]

TABLE 4 **Distribution of Patents According to Number of Citations**

Number of Citations	Number of Patents	Percentage of Patents	Cumulative Percentage
0	215	47.1	47.1
1	78	17.1	64.3
2	54	11.8	76.1
3	35	7.7	83.8
4	21	4.6	88.4
5	10	2.2	90.6
6	15	3.3	93.9
7	8	1.8	95.6
8	3	0.7	96.3
9	3	0.7	96.9
10	2	0.4	97.4
12	1	0.2	97.6
13	2	0.4	98.0
14	1	0.2	98.2
16	1	0.2	98.5
17	2	0.4	98.9
19	1	0.2	99.1
20	1	0.2	99.3
21	1	0.2	99.6
25	1	0.2	99.8
72	1	0.2	100.0

[18] Recall, however, from footnote 17 that the 1972 patent was simply added to the 1973 patents in computing the correlations. Thus, the first lag was actually longer (about one year long), and the overall lag would increase from three to four months if one averages that first lag with the rest.

[19] Campbell and Nieves (1979) present the distribution of citations for all U.S. patents issued from 1971 to 1978, and Narin (1983) does the same for 13,264 chemical and allied product patents issued in 1975. Both distributions look remarkably similar to the one for CT scanners. Unfortunately, the citation values in Campbell and Nieves (1979) only go up to 13+, and therefore, I cannot be sure whether the distribution for CT is typical in its upper tail.

[20] According to Scherer (1965), ". . . patent statistics are likely to measure run-of-the-mill industrial inventive output much more accurately than they reflect the occasional strategic inventions which open up new markets and new technologies." Of course, Scherer was quite right at the time, given the kind of data available then.

☐ **Nonlinearities in citations.** In constructing the weighting index, I have implicitly assumed so far that a citation is worth as much as a patent, i.e., that the weights are linear in the number of citations. However, there may be something akin to increasing or decreasing returns to the informational content of citations, in which case the weighting scheme would be nonlinear. Consider the more general specification

$$WPC_t(\alpha) = \sum_{i=1}^{n_t} (C_i^\alpha + 1) = n_t + \sum_{i=1}^{n_t} C_i^\alpha, \qquad \alpha > 0.$$

Notice that the nonlinearity is assumed to occur in the citations to each patent, rather than in some patent aggregate (e.g., in the yearly counts), since the results would otherwise be highly sensitive to the chosen aggregation scheme, which is rather arbitrary. The problem now is to find α_1^* and α_2^* such that [21]

$$\alpha_1^* = \underset{\alpha}{\text{argmax}} \text{ corr } [WPC_t(\alpha), \Delta W_t] \quad \text{and} \quad \alpha_2^* = \underset{\alpha}{\text{argmax}} \text{ corr } [WPC_t(\alpha), TW_t].$$

As can be seen in Table 5, the maxima occur when $\alpha_1^* = 1.3$ and $\alpha_2^* = 1.1$; that is, the weighting scheme is in fact convex in the number of citations per patent. (Note, however, that the relationship between WPC_t and ΔW_t is still linear.) This finding means that the marginal informational content of WPC_t increases with the number of citations, strengthening the potential role of WPC_t as an indicator of the value of innovations. Furthermore, it implies that the variance in the value of patents is larger and that the distribution of those values is more skewed than what could be inferred from the simple count of citations.

TABLE 5 Correlations[a] of *WPC* with ΔW and *TW*: Searching for Nonlinearities

	Patents to Firms in Computed Tomography				All Patents	
	Contemporary		Lagged Three Months		Lagged Three Months	
Exponent α	ΔW	TW	ΔW	TW	ΔW	TW
0.80	0.455 (0.19)	0.543 (0.10)	0.512 (0.13)	0.653 (0.04)	0.329 (0.35)	0.503 (0.14)
0.90	0.538 (0.11)	0.590 (0.07)	0.601 (0.07)	0.711 (0.02)	0.419 (0.23)	0.570 (0.09)
1.00	0.616 (0.06)	0.626 (0.05)	0.685 (0.03)	0.755 (0.01)	0.513 (0.13)	0.635 (0.05)
1.10	0.680 (0.03)	0.642 (0.05)	0.754 (0.01)	0.777 (0.008)	0.605 (0.06)	0.687 (0.03)
1.20	0.721 (0.02)	0.635 (0.05)	0.798 (0.006)	0.770 (0.009)	0.684 (0.03)	0.719 (0.02)
1.30	**0.738** (0.01)	0.607 (0.06)	**0.813** (0.004)	0.736 (0.02)	0.738 (0.02)	**0.720** (0.02)
1.40	0.730 (0.02)	0.560 (0.09)	0.800 (0.006)	0.677 (0.03)	**0.760** (0.01)	0.689 (0.03)
1.50	0.703 (0.02)	0.501 (0.14)	0.766 (0.01)	0.606 (0.06)	0.751 (0.01)	0.634 (0.05)
1.60	0.663 (0.04)	0.436 (0.21)	0.718 (0.02)	0.527 (0.11)	0.719 (0.02)	0.562 (0.09)

[a] Pearson correlation coefficients.
Significance levels for H_0: corr = 0 are given in parentheses.

[21] Notice that it is not possible to estimate those exponents by, say, running a regression in the logs, since the nonlinearity refers to the citations to each patent.

Notice that the results that $\alpha_i > 1$, $i = 1, 2$, are robust (that is, they occur also when there is no lag and when all patents, rather than patents to firms in CT, are used), and that α_1^* and α_2^* are global maxima. Note also that $\alpha_1^* > \alpha_2^*$; that is, the nonlinearity is stronger when patent counts are related to ΔW rather than to TW. Recall that ΔW is a measure of the gains to the representative consumer, whereas TW multiplies ΔW by the number of consumers that benefit from the innovation at present and in the future. Thus, the fact that $\alpha_1^* > \alpha_2^*$ and that corr $[WPC(\alpha_1^*), \Delta W] >$ corr $[WPC(\alpha_2^*), TW]$, can be taken to mean that citations are more informative of the value of innovations *per se,* rather than of the size of the market for the products embedding those innovations. This is reassuring, since we expect that the factors related to the technology itself (rather than to market size) will be dominant in the citing process.

□ **The second hypothesis.** The relationship between patents and R&D has been intensively scrutinized in past research,[22] and the results appear to be quite uniform, centering around the following stylized facts: (a) there is a strong statistical association between patents and R&D expenditures; (b) this relationship appears to be mostly contemporaneous; and (c) R&D explains a great deal of the cross-sectional variance in patenting but not much of the variation over time. The second hypothesis also postulates a close association between patents and expenditures on R&D, but within a given field over time rather than across firms or industries. From Table 6 we see first that there is indeed a high correlation between SPC_t and $R\&D_t$ and a much weaker one between $R\&D_t$ and WPC_t; the second hypothesis is therefore amply confirmed. Second, the degree of association peaks when $R\&D_t$ is lagged just five months, supporting previous findings of short gestation lags. Third, the correlations are slightly higher for counts of all patents than for patents to firms in CT, suggesting some degree of spillovers from the R&D done by CT manufacturers to other assignees.

It is also worth reporting the following correlations, which indicate that SPC_t tends to move together, not just with R&D, but also with other manifestations of the innovative

TABLE 6 **Correlations[a] Between Patent Counts and R & D**

	All Patents		Patents to Firms in Computed Tomography	
Lags	SPC_t	WPC_t	SPC_t	WPC_t
None	0.869 (0.0002)	0.609 (0.05)	0.843 (0.001)	0.525 (0.097)
Three months	0.919 (0.0001)	0.591 (0.04)	0.912 (0.0001)	0.495 (0.102)
Four months	0.924 (0.0001)	0.582 (0.05)	0.914 (0.0001)	0.491 (0.105)
Five months	**0.933** (0.0001)	0.577 (0.05)	**0.918** (0.0001)	0.483 (0.112)
Six months	0.921 (0.0001)	0.543 (0.07)	0.903 (0.0001)	0.450 (0.142)
One year	0.831 (0.0008)	0.248 (0.44)	0.794 (0.002)	0.152 (0.638)

[a] Pearson correlation coefficients.
Significance levels for H_0: corr = 0 are given in parentheses.

[22] Many of the articles in Griliches (1984) have to do with this issue; extensive references to previous works can also be found there.

action taking place in a given field over time. (All are contemporaneous; the data are taken from Table 2.)

$$\text{corr}\,(SPC_t, \text{Number of Firms in the CT Market}) = \underset{(.0007)}{.858}$$

$$\text{corr}\,(SPC_t, \text{Number of New Scanners Introduced in the Market}) = \underset{(.002)}{.813}$$

$$\text{corr}\,(SPC_t, \text{Number of New Adopters of CT}) = \underset{(.0002)}{.913.}$$

The first two correlations reflect the fact that competition in the CT market was driven primarily by rivalry in innovation, whereas the third has to do with the interaction between innovation and diffusion.

6. The usefulness and meaning of patent data: concluding remarks

■ The findings presented above suggest that patent citations may be indicative of the value of innovations and, if so, that they may hold the key to unlock the wealth of information contained in patent data. In order to understand the various roles that patent-based indicators may thus come to play, it may be helpful to use a familiar analogy, namely, to think of patents as working papers in economics and, accordingly, of economic departments as firms, of fields in economics as industries, and so forth. Working papers are "produced" roughly in proportion to the number of faculty, as patents are with respect to R&D. The fact is that it does not take much to get a patent once the firm has an established R&D facility going, as it does not take much to write a working paper. Still, a larger number of patents presumably indicates that much research efforts have been invested by the R&D staff, as more papers would suggest that the faculty is "trying harder." Thus, a simple patent count could be regarded as a more refined input measure (vis-a-vis R&D), in the sense that it incorporates part of the differences in effort and nets out the influence of luck in the first round of the innovative process. Of course, as with patents, a mere count of working papers written does not say much about the value of the scientific contributions made; for that, one would need information on whether and where they get published, the number of citations that they receive over time, etc. Clearly, those indicators would be to working papers what patent citations are to patents.

Beyond establishing the role of citations as indicators in a purely statistical sense, the results of this article can also be seen as lending support to a particular view of the innovative process, in which context citations are associated with real phenomena rather than just being a useful data contrivance. This view sees innovation as a continuous time process that has a predominantly incremental nature, punctuated by occasional breakthroughs that bring forth subsequent innovative efforts and direct them into novel channels.[23] Congruent with such a view, a patent would be regarded as important if it opened the way to a successful line of further innovations; the patents coming in its wake would naturally cite it, and hence, those citations could be taken as first-hand evidence of the path-breaking nature of the original patent. A particular case is when the important patent(s) refers to a new product (as, say, the first few patents by Hounsfield in CT): truly novel ideas are almost by definition crude and lacking at first but get better over time as a result of further research efforts. These efforts would generate down-the-line patents aimed at refining and improving the original

[23] This view stands in opposition to the innovative process being conceived of as a sequence of discrete, well-compartmentalized, and sizeable events that occur essentially in a random fashion from the point of view of technology itself. Such a view presupposes a one-to-one correspondence between each of those singular innovations and patents, leading to an interpretation of patent importance and of patent citations quite different from the one held here.

innovation and, again, these patents would be most likely to cite the basic one. The key point is that in this context, citing patents would bear a sort of causal relationship to the cited patent, with citations being the overt manifestation (instituted as common practice by the Patent Office) of such a link.

Once their meaning has been well established, the use of patent data may offer additional advantages in itself and over alternative data sources. First, patent data can be easily obtained all the way to the very beginning of a product class, whereas the gathering of conventional industry data usually starts only when a sector is well established. Thus, patent counts and citations may play an important role in studying the very emergence of new products, which seems to be the period when most of the important innovations occur. Second, patent data are richer, finer, and have a wider coverage than say, R&D expenditures, and are practically continuous in time.

All of my conclusions have been expressed in a qualified manner, since they are based upon the findings from a single case study. It is important to emphasize, however, that the sort of validation of the citation-based patent index attempted here could hardly have been done in a wider context simply because the measures of the value of innovations that would be required for that purpose are nowhere to be found. (If such measures were widely available, we would hardly need the more imperfect patent indicators.) I hope that future research along similar lines will bring in more supportive evidence and further demonstrate the attractiveness of the proposed indicators.

Appendix

■ A statistical analysis of truncation and age effects follows.

Testing age versus importance. The starting point for the test is the specification of a hypothetical citation process under which all patents are of equal importance, and hence the only differentiating factor is age. The distribution of citations thus generated could then be compared with the actual one, and the maintained hypothesis that all patents are equally important could be tested with the aid of a Pearson semiparametric χ^2-test. (See DeGroot (1975).) As a first step, patents are ordered according to their application date and indexed with $i = 1, \ldots, N$ ($N = 456$). (Thus, i indicates the cumulative number of patents in CT applied for up to patent i.) Denoting the probability that patent i will be cited in patent j by p_{ij} (for $i < j$) and the number of references to previous patents in CT appearing in patent j by r_j, I define the patents $1 \le i < j$ to be iso-important if

$$p_{ij} = \frac{r_j}{j-1} = p_j, \qquad j = 1, \ldots, N. \tag{A1}$$

Thus, equal importance is taken to mean that all patents applied for up to a certain point in time have the same probability of being cited by a subsequent patent. In other words, (A1) means that the citations appearing in patent j are the result of r_j random drawings (without replacement) from a pool containing the j-1 patents that preceeded it.[24] Noting that (A1) also implies time independence (that is, for any $i < j < k$, p_{ik} is independent of p_{ij}), the expected number of citations of patent i can be computed simply as $C_i^e = E(C_i) = \sum_{j=i+1}^{N} p_j$. Obviously, $C_i^e > C_j^e$ for any $i < j$; that is, older patents will get more citations on average than recent ones, just by virtue of their age. Notice also that p_j has to decrease eventually with j,[25] thus reinforcing the pure age effect. That is, not only do later patents miss the earlier p_j's, but those probabilities tend to be the large ones, a fact that further reduces the expected number of citations of recent patents vis-a-vis older ones.

In order to perform the χ^2-test, the data were aggregated by months, since it would be unreasonable to attach any significance (in the sense of differences in C_i^e) to the precise day of application. Indexing by τ and t the number

[24] Clearly, this is not the only possible definition of iso-importance. Notice, however, that by defining p_{ij} to be independent of the distance $(j - i)$, I implicitly favor the earlier patents, thus increasing the power of the test. That is, any plausible departure from (A1) would have the probabilities decrease with $(j - i)$, making the distribution of expected citations more uniform, and hence, making it easier to reject the null hypothesis.

[25] This must be true unless r_j increases indefinitely over time, which is highly unlikely; in the case of CT, r_j was quite stable over the whole period.

of months elapsed since January 1972, the observed number of citations is $C_t^o = \sum_{i=1}^{n_t} C_i$, where n_t is the number of patents in month t. Similarly, redefining (A1) in monthly terms gives

$$p_\tau = \sum_{i=1}^{n_\tau} r_i / \sum_{j=1}^{\tau-1} n_j, \tag{A2}$$

and so, $C_t^e = n_t \sum_{\tau=t+1}^{T} p_\tau$. Turning now to the test,[26]

$$\chi^2 = \sum_{t=1}^{156} \frac{(C_t^e - C_t^o)^2}{C_t^e} = 1025 >> 147 = \chi^2(110), \quad \text{where} \quad \alpha = .01.$$

Thus, the hypothesis that the observed distribution of citations is due solely to age is strongly rejected. As is to be expected, the largest discrepancies between actual and expected values occur at the very beginning of the period. In particular, the values for the first patent are $C_1^o = 72$, $C_1^e = 5.96$, and hence, $(C_1^e - C_1^o)^2/C_1^e = 731$, which amounts to .75 of the computed χ^2-statistic. Since this first patent can be regarded in many ways as an exception, the test was redone after deleting it; again, the null hypothesis is rejected by a wide margin.

Assessing the truncation bias. The other potential problem in this context is that the (unavoidable) truncation of the data might induce a bias in the citation counts; the extent of such bias, in turn, will depend upon the behavior of citation lags and the rate of new patent arrivals after the date of search. Citation lags refer to the length of time elapsed between the dates of the citing patents and of the cited patent: the shorter they are, the less severe the problem will be.[27] Denote the frequency distribution of citation lags by f_τ; for example, if year t patents are to receive (on average) C_t citations per patent, f_τ stands for the percentage of those citations to be received after τ years. (Thus, $\sum_{\tau=t}^{\infty} f_\tau = 1$.) Likewise, define $c_{t\tau} = f_\tau C_t$ and $g_{t\tau} = c_{t\tau}/n_\tau$, where n_τ is the total number of patents in year τ. Now, suppose that because of truncation, one can actually obtain only a fraction, h_τ, of them; then, assuming that $g_{t\tau}$ is invariant with respect to h_τ (i.e., that citations of year t patents are randomly distributed among the n_τ patents), the observed average number of citations to year t patents will be $c_{t\tau}^o = g_{t\tau} h_\tau n_\tau = h_\tau f_\tau C_t$. Thus, given the sequences $\{h_\tau, f_\tau\}$, one can compute for each year the fraction $v_t = \sum_{\tau=t}^{\infty} h_\tau f_\tau$, where v_t stands for the percentage of citations that patents in year t can be expected to receive out of the total that they would have received had it not been for the truncation of the data. Using the granting-application lags to obtain h_τ and the citation lags for f_τ, the expected biases are easily computed and presented in Table A1.

Thus, we do miss a few citations because of the truncation of the data; moreover, there is, as expected, a truncation bias in the sense that we have a smaller fraction of the true number of citations of later patents than of earlier ones. However, the absolute expected number of missing citations is very small, and hence, it is clear that

TABLE A1 **Expected Biases**

		Number of Citations		
Year of Cited Patents	v_t	Actual	Missing (Rounded)	Fraction Missing
Up to 75	1.000	491	0	0.00
76	0.998	169	0	0.00
77	0.990	145	1	0.01
78	0.969	55	2	0.04
79	0.930	29	2	0.07
80	0.861	7	1	0.14
81	0.732	3	1	0.33
82	0.527	1	1	1.00

[26] The summation is done in principle over 156 months, covering the thirteen years from 1972 to 1984. However, patents were actually applied for only in 111 months out of the 156, and hence, there are just 110 degrees of freedom (d). When d exceeds 100 (as it does here), the critical χ^2 value is to be computed as follows (see Harnett (1975)): $\chi^2 = 1/2(z_\alpha + \sqrt{2d - 1})^2$, where z is the standardized normal deviate (for $\alpha = .01$, $z_\alpha = 2.3263$).

[27] In the present case, these lags are relatively short (the mean lag is three years), suggesting that the truncation problem is not too severe on that account. For a detailed discussion, see Trajtenberg (1990).

the truncation problem cannot possibly affect the conclusions of this article. (This holds true even if the bias had been for some reason underestimated by, say, a factor of two.)

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